

# MICs IN A MULTILAYER DIELECTRIC MEDIA IN A RECTANGULAR WAVEGUIDE

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## Abstract

This paper describes a full-wave analysis of MICs in a multilayer dielectric media in a rectangular waveguide. The analysis combines the Spectral Domain approach with residue theory and the contour integration method to accurately evaluate the impedance matrix of the method of moments.

## 1 INTRODUCTION

A variety of techniques have been used to analyze Microwave Integrated Circuits (MICs) [1]. Although very accurate, application of full-wave techniques using Method of Moments (MoM) are often time consuming. Recently an effort has been made to overcome this difficulty, allowing for such an analysis to be quickly performed on CAD packages. For example, when analyzing boxed MICs, 2-D FFT algorithms have been used [3] to evaluate the doubly infinite summations in the moment matrix. It makes the SDA very efficient but limited to structures with identical segment dimensions. In this work we present a full-wave analysis of planar MIC in a multilayer dielectric media in a rectangular waveguide. It uses the Spectral Domain approach and the elements of moment matrix are evaluated by subtracting a conveniently

modified asymptotic limit of its integrand. The integral of the limit is obtained using residue theory and the contour integration method, resulting in a faster and more accurate procedure. The Matrix Pencil technique [4] is used later for obtaining the S-parameters of the given circuit.

## 2 FORMULATION

The cross-section of the waveguide is shown in Figure 1. All metals are assumed to be perfect conductors, and a time variation of  $e^{j\omega t}$  is assumed and suppressed. Each layer consists of a homogeneous, isotropic and lossless dielectric characterized by its permittivity ( $\epsilon_i$ ) and permeability ( $\mu_i$ ).

Suppose that an arbitrary surface current distribution exists at the dielectric interfaces. The electromagnetic fields in each layer can be expressed in terms of the  $y$ -components of the vector potentials ( $A_y$  and  $F_y$ ). The Green's functions for the vector potentials are obtained by imposing the boundary conditions at the dielectric interfaces. The integral equation of the problem arises from the the boundary conditions on the surface of the circuit (S), supposing the presence of an incident field  $E^{in}$ . The currents are expanded in rooftop basis functions which are also used as weighting functions, resulting in a Galerkin formulation.

The first numerical problem arises from the poles of the integrand, and it is overcome by subtracting the singularity from the integrand, and analytically evaluating the remaining singular integral. The second and more troublesome problem when evaluating the elements of matrix  $[Z]$  is due to the slow convergence of the summation and integrals, specially for the case when source and testing functions are in the same interface. The evaluation of the impedance matrix becomes computationally expensive, and its accuracy is endangered by the oscillatory nature of the integrands. To overcome this limitation, the limit of the integrand, as  $\sqrt{k_z^2 + \alpha_n^2}$  goes to infinity, is added and subtracted to the original integrand:

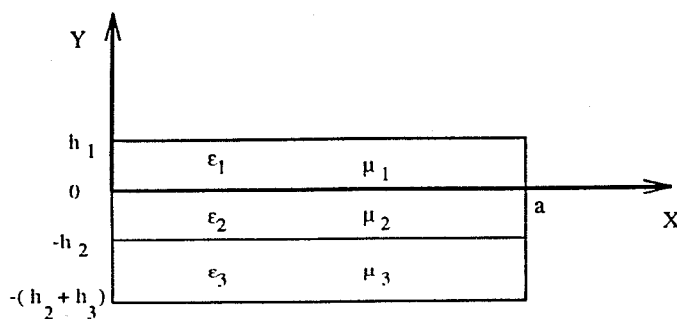


Figure 1: Cross-section of the Waveguide.

$$Z_{zzm_fm_s} = \frac{a}{4\pi} \sum_{n=1}^{\infty} \left[ \int_0^{\infty} \left( Int_{zz}^+ - \lim_{\sqrt{k_z^2 + \alpha_n^2} \rightarrow \infty} Int_{zz}^+ \right) dk_z + L_{zzm_fm_s} \right] \quad (1)$$

where:

$$L_{zzm_fm_s} = \int_{-\infty}^{\infty} \lim_{\sqrt{k_z^2 + \alpha_n^2} \rightarrow \infty} Int_{zz} dk_z \quad (2)$$

Now, the integral in  $Z_{zzm_fm_s}$  can be evaluated quickly as the integrand converges asymptotically to zero. The convergence problems of  $Z_{zz}$  have been isolated in  $L_{zzm_fm_s}$ , which will be evaluated analytically. A basic procedure for evaluating integrals on the real axis, is to close the contour with a semi-circle of infinity radius ( $C_{\infty}$ ), and to use the residue theory. In our case, the limit above has two branch-cuts, with branch-points at  $k_z = \pm j\alpha_n$ , first order poles at  $k_z = \pm j\alpha_n$ , and a higher order pole at  $k_z = 0$ . Geometrically the limiting case represents a multilayer media with the layers 1 and 3 extending to  $y = \infty$  and  $-\infty$ , respectively. Those branch cuts account for the corresponding radiation in this now open structure. But physically they have no meaning to the original problem. We can avoid the branch-cut integrations and the higher order pole by changing the limit of the integrand ( $Int_{zz}$  and  $Int_{zz}^+$ ) according to the following substitutions:

$$\frac{1}{\sqrt{k_z^2 + \alpha_n^2}} \rightarrow \frac{\tanh(\sqrt{k_z^2 + \alpha_n^2})}{\sqrt{k_z^2 + \alpha_n^2}} \quad (3)$$

$$\frac{1}{k_z^2} \rightarrow \frac{1}{k_z^2 + (r\pi/a)^2} \quad r < 1 \quad (4)$$

The integration around the branch cut is substituted by a summation of residues, and the higher order pole at  $k_z = 0$  is substituted by first order poles at  $k_z = \pm jr\pi/a$ . It should be stressed that these modifications do not alter the limit, as the hyperbolic tangent converges rapidly to 1, and  $r < 1$ .  $L_{zz}$  (2) can now be written as a combination of the integrals which are easily evaluated by contour integration and residue theorem. As a result  $L_{zz}$  is expressed by exponentially decaying summations unless  $d = 0$ . In this case the summations need to be evaluated only once, and their values stored for future reference. A very similar procedure is used for the elements of the submatrices  $Z_{xx}$  and  $Z_{zz}$ .

The advantage of the procedure described is to limit the amount of numerical integration performed by drastically improving the behavior of the integrand. Furthermore, the limit of the integrand is reached when  $\sqrt{\alpha_n^2 + k_z^2}$  goes to  $\infty$ . This means that as we increase  $n$  in the summation of the elements of matrix  $[Z]$  the numerical integration needs to be performed over a decreasing interval length. The overall effect is a combination of an increase in the accuracy of the elements of the impedance matrix, and substantial savings in computational time spent to fill out the matrix  $[Z]$ .

### 3 EXAMPLES

As an example consider the a microstrip gap with overlay half-wave resonator as shown in Figure 2. The parasitic resonator is placed symmetrically over the gap, with the same width of the transmission line. The magnitude of the reflection and transmission coefficients are shown in Figure 3, together with numerical results and measurements obtained by Yeung [5] for the case of an open multilayered structure. It is observed that the resonance frequency shifts slightly towards 4 GHz.

As a second example consider the coupled microstrip filter shown in Figure 4. The magnitude of the transmission coefficient is shown in Figure 5. It is compared with measurements and theoretical results obtained by Shibata [6] for the open microstrip case, and with theoretical results for the filter inside a cavity, obtained by Railton [3]. It can

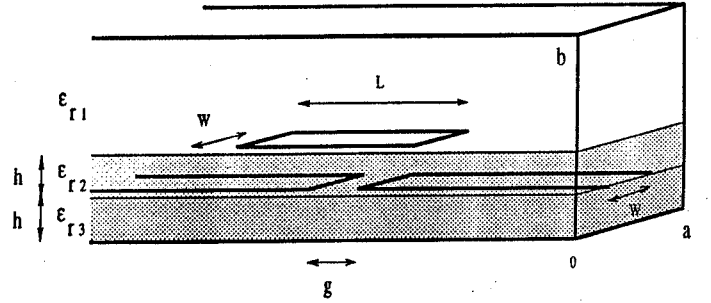


Figure 2: Microstrip gap with overlay half-wave resonator.

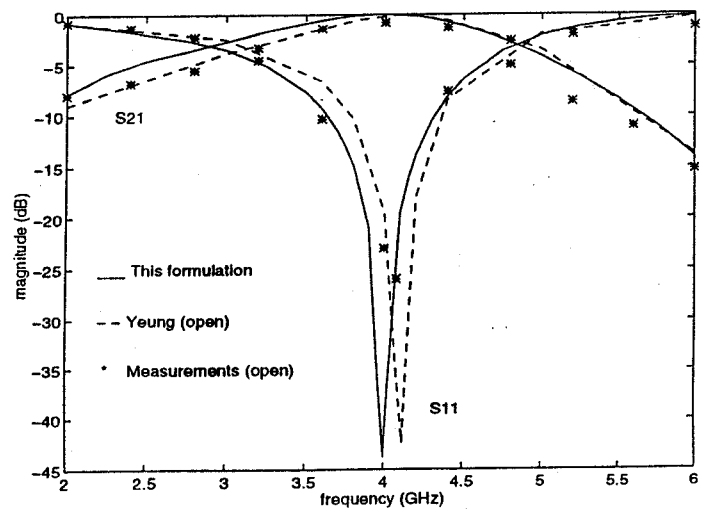


Figure 3: Magnitude of S11 and S21 for the Microstrip gap with overlay resonator.  $a = b = 20$  mm,  $h = 0.8382$  mm,  $\epsilon_{r1} = 1$ ,  $\epsilon_{r2} = \epsilon_{r3} = 2.33$ ,  $w = 2.3$  mm,  $g = 1.0$  mm,  $L = 27.3$  mm.

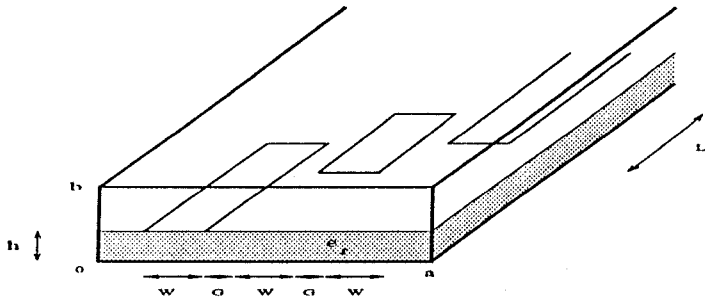


Figure 4: Coupled Microstrip Filter.

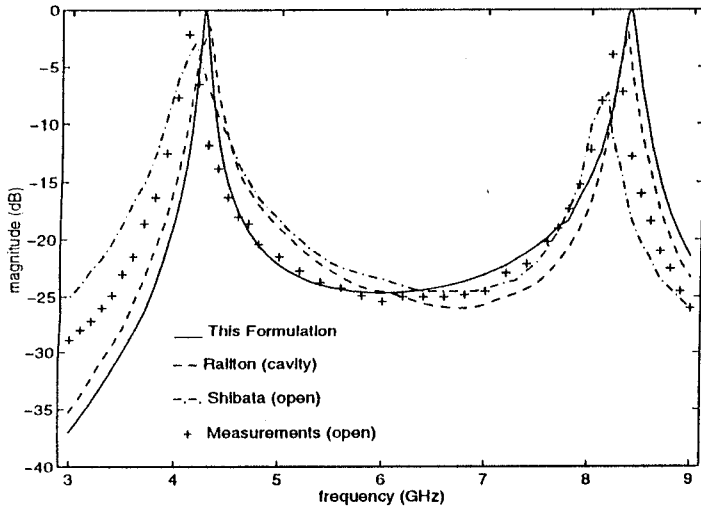


Figure 5: Magnitude of S21 for the Coupled Microstrip Filter.  $a = 11.62$  mm,  $b = 16$  mm,  $h = w = G = 1.272$  mm,  $\epsilon_r = 10$ ,  $L = 12.72$  mm.

be noted a shift of 0.1 GHz between the shielded cases and the open one.

Figure 6 shows an example of coupling between two transmission lines at different dielectric interfaces, overlapping for a distance  $d$ . Figure 7 shows the magnitude of the transmission coefficient as a function of  $d$  (negative  $d$  means that the lines are apart) at a frequency of 10 GHz. The results are compared with those obtained by Schwab and Menzel [7] for the case when the coupling is housed in a rectangular cavity. There is a very good agreement, and the coupling presents a wide range depending on the overlapping distance  $d$ .

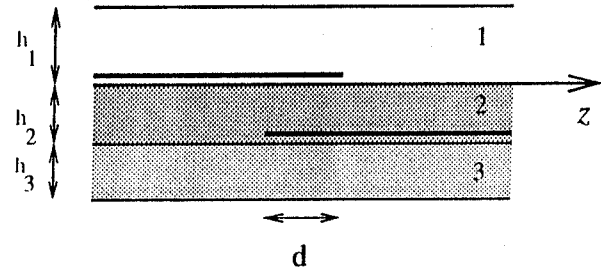


Figure 6: Transmission Line Coupling.

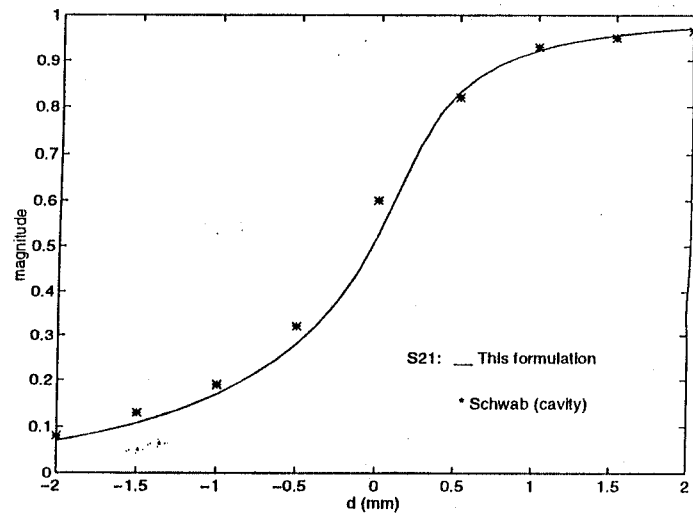


Figure 7: Magnitude of S21 for the Transmission Line Coupling.  $a = b = 5$  mm,  $h_1 = h_3 = 2.373$  mm,  $h_2 = 0.254$  mm,  $\epsilon_{r1} = \epsilon_{r3} = 1$ ,  $\epsilon_{r2} = 2.2$ ,  $w = 1.75$  mm,  $f = 10$  GHz.

## 4 CONCLUSION

This work has presented a full-wave analysis of MIC in multilayer dielectric media in a rectangular waveguide. It shows a procedure that permits an accurate and faster numerical evaluation of the moment matrix, without the limitations of FFT routines, as previously described. Results have been compared with previously published data and a good agreement is observed.

## References

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